16 July 2012

Macroeconomics' WTI Crude Oil Forecasting Model

In the event of any enquiries please contact:

Laura Ryan

T: 02 6161 3542

E: laura.ryan@macroeconomics.com.au

Table of Contents

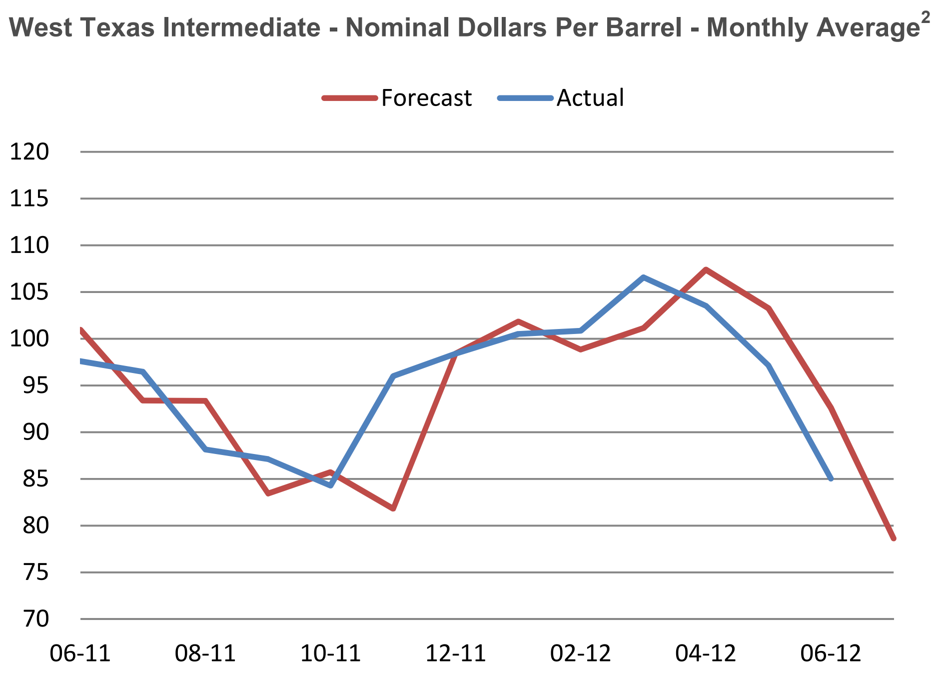
About the Author

Laura Ryan is the Manager of Quantitative Analysis at Macroeconomics based in Canberra. She is responsible for the statistical analysis of financial markets undertaken by us. She is also a Lecturer at the Australian National University in the school of Finance and Applied Statistics where she has taught units in financial statistics, business statistics, and statistics for the life sciences. Laura's research focus is on Subset Vector Autoregression and model selection techniques in the context of improved forecasting. Previously Laura has worked at Commonwealth Bank as a Quantitative Strategist where she was responsible for the quantitative strategy and research for the CBA fixed income, FOREX and Equities clients. More recently, Laura held the role of Quantitative Manager at AMP Capital Investors. In this role she applied quantitative techniques to price and hedge domestic and international derivative transactions and manage existing funds for the Structured Products Group.

Executive Summary

The **Macroeconomics** Oil Forecasting Model is updated monthly and provides a one month ahead forecast for the West Texas Intermediate Crude Oil Price using relative oil inventories. Oil inventories are widely accepted as the most important predictor of world oil prices. Oil inventories reflect the demand and supply imbalances that drive fluctuations in the oil price. The global oil price is modelled using an implementation of Subset Autoregression with Exogenous Variables (SARX). Coefficient and standard error estimates obtained from SARX have traditionally been determined by conditioning on a single "best model". Estimates from a single model ignore model uncertainty and result in under-estimated standard errors and over-estimated coefficients. Macroeconomics utilises a special modeling technique which allows for model uncertainty and therefore leads to outperformance when compared to forecasts made from a single model. The model is dynamic and will evolve in structure with continued research into the factors that drive the oil market. The monthly forecast is available as a publication which can be purchased from our website: **http://macroeconomics.com.au/publications**.[1] The model was developed by Laura Ryan, the Manager of Quantitative Analysis at **Macroeconomics** based in Canberra and is updated at the end of every calendar month.

[1] For inquires about how Macroecomics can assist with any modeling your business undertakes, please contact Laura Ryan on (02) 6161 3542.



1 Month Ahead Forecast - Dollars Per Barrel

Date	Forecast	Actual	Residual
31/07/2012	78.61	-	-
29/06/2012	92.51	85.04	-7.47
31/05/2012	101.61	97.11	-4.50
30/04/2012	107.11	103.53	-3.58
30/03/2012	100.79	106.56	5.77
29/02/2012	98.51	100.85	2.34
31/01/2012	102.39	100.50	-1.89
30/12/2011	98.5	98.40	-0.10
30/11/2011	82.87	95.97	13.10
31/10/2011	85.8	84.27	-1.53
30/09/2011	83.87	87.12	3.24
31/08/2011	93.29	88.13	-5.16
29/07/2011	94.09	96.46	2.37
30/06/2011	100.71	97.58	-3.13

[2] The Greek Debt Crisis was in full swing during 2011. The VIX was up around 48 in August 2011 and maintained a level above 30 till the beginning of December 2012.

Introduction

We model the one month ahead West Texas Intermediate (WTI) crude oil price using relative oil inventories. Oil inventories are widely accepted as the most important predictor of world oil prices. Oil inventories reflect the demand and supply imbalances that drive fluctuations in the oil price. See (Kilian and Murphy 2011; Pang, Xu, Yu, Ma, Lai, Wang, and Xu 2011; He, Wang, and Lai 2010; Manera, Longo, Markandya, and Scarpa 2007; Ye, Zyren, and Shore 2006, 2005; Merino and Ortiz 2005; Ye, Zyren, and Shore 2002). The model is dynamic and will evolve in structure as new research into the variables that drive the market are identified (Roberts and Ryan 2012).

Although similar models have been applied to the oil market in the past (Ye et al. 2002, 2005; Askari and Krichene 2010), ours is the first to allow for the uncertainty surrounding model selection and parameter estimation. Allowing for model uncertainty results in superior forecasting results (Emery and Ryan 2012). When fitting the AR, we allow for the uncertainty encountered during the model selection process and the impact this has on coefficient estimates and their estimated standard errors.

Model uncertainty typically occurs as we only have a single realisation of the data generating process to analyse, so any model fitted may only be capturing characteristics specific to the single sample path and may not generalise to the population. This leads to downwardly biased standard error estimates, over-estimation of coefficients and therefore over-estimation of the importance of the variables themselves.

Further, when we attempt to model a large number of variables and therefore consider a large set of initial candidate models, model uncertainty is exacerbated (Miller 1990; Chatfield 1995; Zucchini 2000; Anderson and Burnham 2003). A number of approaches have been developed that attempt to deal with the model uncertainty problem. Among them are: Bootstrap model averaging (Buckland, Burnham, and Augustin 1997), Bayesian model averaging (Clyde 2000), Bootstrap estimation of coefficients and confidence intervals (Austin 2008), and Multi-model inference (Anderson and Burnham 2003).

We use multi model inferences to forecast the one month ahead WTI price. Multi model inferences are shown to have better predictive performance than relying on a single winning model. See (Eicher, Papageorgiou, and Raftery 2011) for a discussion on improved predictive performance and (Lavou and Droz 2009) for practical implementations of multi model inferencing using the bootstrap and Bayesian model averaging. (Anderson and Burnham 2003), (Anderson and Burnham 2004) explain a multi model inferencing approach that utilises

Akaike weights after first identifying a small a-priori set of candidate models. We follow the approach as specified by (Anderson and Burnham 2004).

Data

We model the monthly average nominal West Texas Intermediate (WTI) crude oil spot prices using the following time series.

Variable	Description	Units	Frequency	Source
$y(t)$	West Texas Intermediate (WTI) crude oil spot monthly average	Dollars per barrel	Daily	Thompson Reuters Datastream
$w(t)$	OECD End-of-period Commercial Petroleum Inventory[3]	Millions of barrels	Monthly	US Energy Information Administration (EIA)

The monthly average WTI spot price is an endogenous variable as represented by $y(t)$. We model two exogenous variables, relative petroleum inventories as represented by $z(t)$ and the 1 month change in inventories as represented by $dw(t)$. As detailed in (Ye 2006a) and (Ye-2005), petroleum inventories exhibit seasonality which must be accounted for when determining relative inventories.

The relative inventory variable at time t is calculated as:

$$z(t) = w(t) - w^{'}(t) \quad t = 1,...,T \tag{1}$$

where $w(t)$ is the actual inventory at time t and $w^{'}(t)$ is the normal inventory level. The normal inventory level as defined in (Ye 2005) is determined as:

[3] While demand for oil is dominated by the US at number one the list at 19.15 million barrels per day as compared to China at 9.06 million barrels per day, we recognise that China will soon need to be recognised in our petroleum inventory series. As of yet, there is no reliable up to date inventory data that suitably represents the demand and supply changes of China. The OECD represents 60% of the world's demand for oil. **http://www.bp.com/sectiongenericarticle800.do?categoryId=9037170&contentId=7068610**. Please also note that the EIA may at times revise their historical petroleum inventory series. This will necessarily mean that there may be revisions of previously reported out of sample forecasts and current out of sample forecasts. Any revisions will be clearly highlighted.

$$w'(t) = \alpha_0 + b_1 T + \sum_{k=2}^{12} b_k D_k + v(t);\, k = 2,...,12 \tag{2}$$

where D_k are eleven seasonal dummies, T is the linear trend and $v(t)$ is the error term.

Model

Given our endogenous and exogenous daily time series $y(t)$, $dw(t)$ and $z(t)$,we now employ a model fitting technique known as Subset Autoregression with Exogenous variables (SARX) to model the global oil market. Full order ARX models are modified to arrive at Subset ARX (SARX). A SARX is a special form of the more basic Autoregression (AR) model and allows us to capture the dynamic behaviour present in the data. By applying AR models to time series data for financial variables, we attempt to identify and quantify the strength of relationships between variables. When using AR models, the data drives the model and the resulting model identifies both the short run and long run influences of variables on other variables, as well as identifying the lags/periods that are significant. Modelling the dynamic interrelationships between variables of interest is a better approach than is possible with other more static descriptive statistics. The assumption that relationships should only be assessed on a static basis ignores the time value of information priced into markets as well as other market constraints. That is, by limiting our analysis to daily co-movements, we are ignoring the fact that often shocks in one market take time to affect another market due to a number of factors. Among these factors are the time it takes for information to flow, liquidity constraints and regulatory constraints. All these factors limit an investor's ability to react immediately to market shocks.

1.1 Full Order Unrestricted Autoregression With Exogenous Variables

Begin with a linear system of the unrestricted reduced form, Autoregression with Exogenous Variables (ARX):

$$y(t) = \alpha_1 + \sum_{i=1}^{m} b_i y(t-i) + \sum_{j=0}^{r} b_{m+j+1} z(t-j) + \sum_{l=1}^{n} b_{l+m+r+1} dw(t-l) + v(t);\, t = 1,...,T \tag{3}$$

with

$$E[v(t)] = 0, E[v(t)v(t)'] = \Omega \tag{4}$$

where: $y(t)$ are observations on the endogenous variables at time t and the $dw(t)$ and $z(t)$ are (unmodelled) observations on the exogenous variables at time t. α is the intercept and $v(t)$ is the error term. $b_\tau, \tau = 1,2,..., m$, $b_\tau, \tau = m+1, m+2,..., m+r+1$, $b_\tau, \tau = m+r+2,..., n$ and denote the coefficients.

The following steps help to identify the optimum model: Select a maximum lag (M) that ensures the true order of the model (m,n,r) is captured. i.e we need $max(m,n,r) \leq M$. (Ye 2006a) find M to be 3 after a series of residual tests. Following determination of m , n and r , estimates of the b_τ are determined assuming that every b_τ is non-zero. This assumption means that no subsets are considered and therefore the possibility of zero coefficients in (1) is also not considered. However, if we are to model economic relationships appropriately then we need to allow for the possibility of entire lags or coefficients being zero. Overparameterisation in the ARX models can also result in poor ex-ante forecasting performance. As a result, we use a structure that allows entire lags to be excluded (Subset ARX).

1.2 Subset Vector Autoregression With Exogenous Variables

Full order ARX models are modified to arrive at Subset ARX (SARX). Given that subsets are allowed under this structure, there exists a large number of possible models that could be estimated. In fact $2^{m+n+r+1}$ models are possible given the structure of (3), where $m+n+r+1$ is the number of parameters per equation.

SARX are determined by imposing restrictions on a set of the original ARX coefficients. In some instances economic theory may help to determine what the restrictions should be. We take a more data driven approach given our limited a *priori* knowledge of what restrictions should be imposed. Our search begins with an unrestricted reduced form of model (1). We change notation slightly to reflect the fact that in the model selection process, quantities defined previously will also depend on the model order q and some lags will be deleted from the full order model. For instance, b_t will now be written as $B_{k,t,I(s)}$ to reflect the dependence on the order of the fitted scheme, where $I(s) = \{i_1,\dots,i_s\} \subseteq \{1,\dots,q\}, 1 \leq i_1 < \dots < i_s \leq q$ represents the included lags. The multivariate SARX(m,r) with included lags $I(s)$ is then represented as:

$$y(t) = \alpha_1 + \sum_{i=1}^{m} B_{i,I(s)} y(t-i) + \sum_{j=0}^{r} B_{m+j+1,I(s)} z(t-j) + \sum_{l=1}^{n} B_{l+m+r+1,I(s)} dw(t-l) + v(t); t = 1,...,T \quad (5)$$

We further modify the the formulation above as in (Ye 2006a) by adding dummy variables to account for the volatility in the oil market resulting from the US 9/11 terrorist attacks and the OPEC quota tightening in April 1999:

$$y(t) = \alpha_1 + \sum_{i=1}^{m} B_{i,I(s)} y(t-i) + \sum_{j=0}^{r} B_{m+j+1,I(s)} z(t-j) + \sum_{l=1}^{n} B_{l+m+r+1,I(s)} dw(t-l) + cApr99 + \sum_{l=0}^{5} d_l I911_j + v(t); t = 1,...,T \quad (6)$$

where c is the coefficient for the level shifting indicator variable $Apr99$ and d_l are the coefficients for the 6 indicator variables $I911_j : j = 0,...,5$ corresponding to the 6 months between October 2001 and March 2002.

Method – Accounting for Model Uncertainty

Traditionally, any model selection process implemented will identify a single ``best" model from a set of candidate models. This means model uncertainty is ignored. To account for model uncertainty we use an information theoretic (IT) approach, based on Akaike weights to identify the most likely models given the data set. We then use these weights to determine relative importance of predictors, calculate model averaged coefficients and then finally make forecasts for the global oil market.

We first briefly explain the Akaike information criterion (AIC) and it's application to model selection. The AIC is defined as:

$$AIC = 2K - 2ln(L)$$

(7)

where K is the total number of parameters estimated in the model and L is the maximum value of the likelihood function for the model given the data. The model with minimum AIC value is chosen as the best model to fit the data. If the errors are assumed to be normally distributed then we can write:

$$AIC = 2K - n\log\left(\frac{SSE}{n}\right) \quad (8)$$

where SSE is the sum of squared errors from the fitted model.

The adjusted AIC shown below should be used where the number of data points is small relative to the number of parameters modelled. $AICc$ converges to AIC as n tend to infinity, and so $AICc$ can be used in cases where the number of data points is also large relative to the number of data points. We use $AICc$ in this study.

$$AICc = AIC + \left(\frac{2K(K+1)}{n-K-1} \right) \tag{9}$$

Define the $AICc$ differences δ_i as:

$$\delta_i = AICc_i - AICc_{min} \tag{10}$$

where $AICc_{min}$ is the model with the smallest $AICc$ amongst the set of models considered and $AICc_i$ is the $AICc$ for model i. (Burnham 2003) present the following rules of thumb:

Table 2: Level of Empirical Support of Model

δ_i	**Support for Model i**
0-2	Substantial
4-7	Considerably Less
> 10	Essentially None

In the analysis to follow we will restrict the set of models to be used as part of our forecasting model to those with a $\delta_i < 2$. We can then calculate the relative likelihood of a model given the data and the set of R models as

$$w_i = \frac{exp(-\frac{1}{2}\delta_i)}{\sum_{r=1}^{R} exp(-\frac{1}{2}\delta_r)} \tag{11}$$

The $w_i's$ are known as Akaike weights and sum to 1 over the set of R models. These weights provide a measure of relative importance which allow us to rank models and variables and calculate model averaged coefficients. We estimate the relative importance of predictors by summing the Akaike weights across all R models. The higher the sum, the relatively more important that predictor is compared to the other predictors considered.

Full model averaged coefficient estimates are calculated as:

$$\overline{B}_i = \sum_{r=1}^{R} w_r I_i(g_r) \hat{B}_{i,r} \tag{12}$$

We prefer the "full average" as opposed to the "subset average". The full average assumes that a variable is included in every model, but in some models the corresponding coefficient is set to zero. The "subset average", does not properly account for model uncertainty and will yield coefficient estimates that are biased away from zero. See (Burnham 2003).

Model Averaged Coefficients

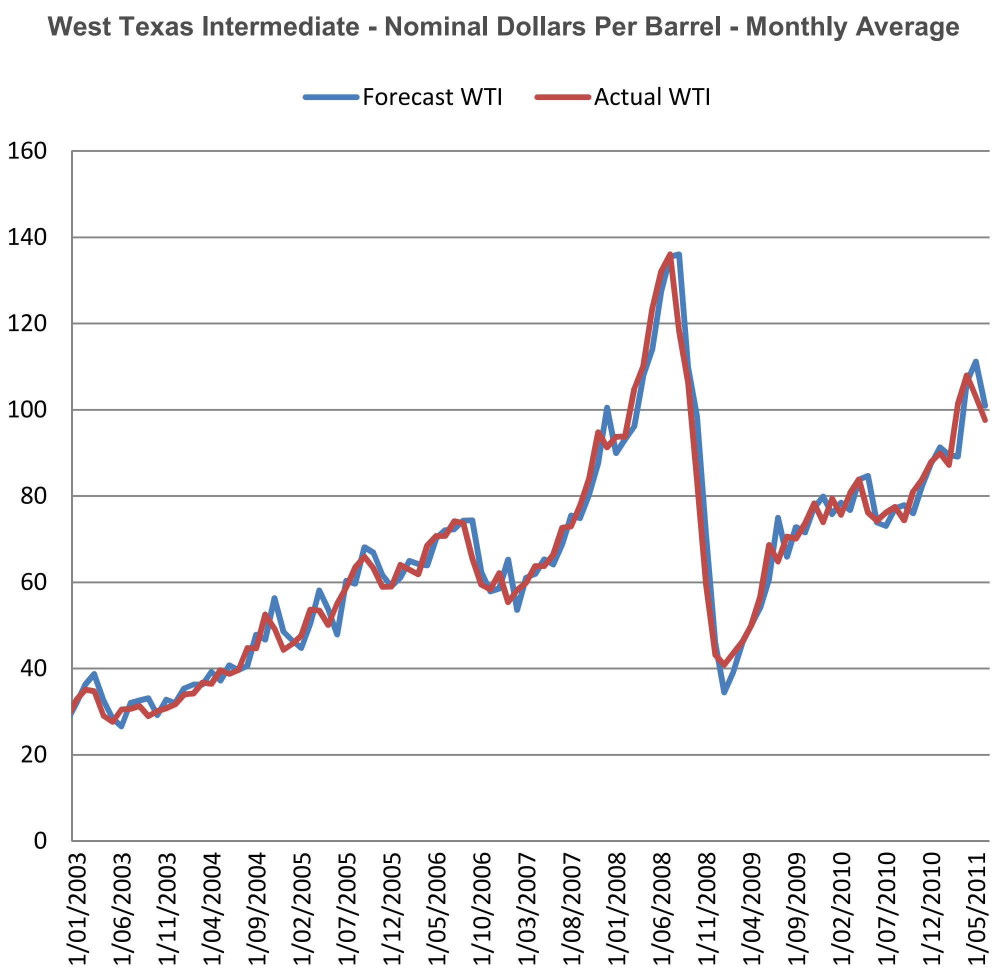
Table 3 presents the most recent set of model average coefficients for the set of models found to have $delta_i < 2$. Note that these rankings and coefficient estimates are dynamic and will be updated at the end of every month.

Table 3: Results

Variable	Description	Coefficient	Relative Variable Importance
	(Intercept)	0.539	1
y(t-1)	Oil Price 1 Month Lag	1.387	1
y(t-2)	Oil Price 2 Month Lag	-0.273	1
y(t-3)	Oil Price 3 Month Lag	-0.144	1
dw(t-2)	Change in inventory level 2 month lag	-0.030	1
cApr99	April Shift Indicator	1.335	0.92
dw(t-1)	Change in inventory level 1 month lag	0.012	0.78
z(t-2)	Relative Inventory 2 Month Lag	0.009	0.50
z(t-3)	Relative Inventory 3 Month Lag	-0.005	0.39
z(t-1)	Relative Inventory 1 Month Lag	-0.001	0.23
dw(t)	Change in inventory level	0.000	0.07

Canberra (Head Office) Phone (within Australia) 02 6161 3542 (outside Australia) +61 2 6161 3542 **Fax** 02 6169 3042 **Melbourne** Phone 03 9583 2142
Postal Address GPO Box 2483, Canberra ACT 2601 **Email** office.manager@macroeconomics.com.au **Web** www.macroeconomics.com.au

In Sample Performance



Canberra (Head Office) Phone (within Australia) 02 6161 3542 (outside Australia) +61 2 6161 3542 **Fax** 02 6169 3042 **Melbourne** Phone 03 9583 2142
Postal Address GPO Box 2483, Canberra ACT 2601 **Email** office.manager@macroeconomics.com.au **Web** www.macroeconomics.com.au

Diagnostics

RMSE	MAE	Hit Rate
3.67	2.58	0.58

In Sample Performance

West Texas Intermediate crude oil spot			
Date	Forecast	Actual	Residual
30/06/2011	100.96	97.58	3.38
31/05/2011	111.20	103.02	8.19
29/04/2011	106.46	108.02	-1.57
31/03/2011	89.12	101.34	-12.21
28/02/2011	89.48	87.18	2.30
31/01/2011	91.26	89.89	1.37
31/12/2010	87.40	87.89	-0.49
30/11/2010	82.40	83.88	-1.48
29/10/2010	76.01	81.05	-5.04
30/09/2010	77.86	74.41	3.45
31/08/2010	77.10	77.51	-0.41
30/07/2010	73.02	76.14	-3.12
30/06/2010	73.84	74.33	-0.49
31/05/2010	84.67	76.19	8.49
30/04/2010	83.76	83.84	-0.07
31/03/2010	76.76	80.71	-3.96
26/02/2010	78.46	75.62	2.84
29/01/2010	75.79	79.38	-3.60
31/12/2009	79.88	73.93	5.94
30/11/2009	77.24	78.30	-1.05
30/10/2009	71.55	73.70	-2.15
30/09/2009	72.80	70.16	2.64

References

Anderson, D. and Burnham, K. (2003). Model Selection and Multi-Model Inference. Springer.

Anderson, D. and Burnham, K. (2004). "Multimodel Inference Understanding AIC and BIC in Model Selection." Sociological Methods Research 33, pp. 261–304.

Anderson, D., Burnham, K., Gould, W., and Cherry, S. (2001). "Concerns about Finding Effects That Are Actually Spurious." Wildlife Society Bulletin 29, pp. 311–316.

Askari, H. and Krichene, N. (2010). "An oil demand and supply model incorporating monetary policy."Energy 35, pp. 2013–2021.

Austin, P. (2008). "Using the Bootstrap to improve estimation and confidence intervals for regression coefficients selected using backwards variable elimination." Statistics in Medicine 27, pp. 3286–3300.

Bernanke, B., Gertler, M., and Watson, M. (2004). "Reply: Oil Shocks and Aggregate Macroeconomic Behavior: The Role of Monetary Policy." Money, Credit and Banking 36, pp. 287–291.

Buckland, S., Burnham, K., and Augustin, N. (1997). "Model selection: an integral part of inference." Money, Credit and Banking 53, pp. 603–618.

Chatfield, C. (1995). "Model Uncertainty, Data Mining and Statistical Inference." the Royal Statistical Society 158, pp. 419–466.

Clyde, M. (2000). "Model uncertainty and health effect studies for particulate matter." Environmetrics 11, pp. 745–763.

Clyde, M. and George, E. (2004). "Model Uncertainty." Statistical Science 19, pp. 81–94.

Eicher, T., Papageorgiou, C., and Raftery, A. (2011). "Default priors and predictive performance in Bayesian model averaging, with application to growth determinants." Applied Econometrics 26, pp. 30–55.

Ryan, Laura and Emery, Martin, Forecasting the West Texas Intermediate Crude Oil Spot Price Using Multi Model Inference (working paper June 6, 2012). Available at SSRN: http://ssrn.com/abstract=2079341

He, Y., Wang, S., and Lai, K. K. (2010). "Global economic activity and crude oil prices: A cointegration analysis." Energy Economics 32, pp. 868 – 876.

Kilian, L. and Murphy, D. (2011). "The Role of Inventories and Speculative Trading in the Global Market for Crude Oil." Working Paper www-personal.umich.edu/ lkilian/MS2010126R1.pdf.

Lavou, J. and Droz, P. (2009). "Multimodel Inference and Multimodel Averaging in Empirical Modeling of Occupational Exposure Levels." Annals of Occupational Hygiene 53, pp. 173–180.

Manera, M., Longo, C., Markandya, A., and Scarpa, E. (2007). "Evaluating the Empirical Performance of Alternative Econometric Models for Oil Price Forecasting." SSRN eLibrary .

Merino, A. and Ortiz, A. (2005). "Explaining the so-called "price premium" in oil markets." OPEC Review 29, pp. 133–152. URL: http://ideas.repec.org/a/bla/opecrv/v29y2005i2p133-152.html

Miller, A. (1990). Subset selection in regression. Chapman and Hall, London.

Roberts, D and Ryan, L, Determining New Oil Market Predictors Under Model Uncertainty (working paper: June 6, 2012). Available at SSRN: http://ssrn.com/abstract=2079348

Ye, M., Zyren, J., and Shore, J. (2002). "Forecasting crude oil spot price using OECD petroleum inventory levels." International Advances in Economic Research 8, pp. 324–333.

Ye, M., Zyren, J., and Shore, J. (2005). "A monthly crude oil spot price forecasting model using relative inventories." International Journal of Forecasting 21, pp. 491–501.

Ye, M., Zyren, J., and Shore, J. (2006a). "Forecasting short-run crude oil price using high- and low inventory variables." Energy Policy 34, pp. 2736–2743.

Ye, M., Zyren, J., and Shore, J. (2006b). "Short-Run Crude Oil Price and Surplus Production Capacity."International Advances in Economic Research 12, pp. 390–394.

Zucchini, W. (2000). "An introduction to model selection." Mathematical Psychology 44, pp. 41–61.