Systems dynamics on the road to zero emissions: a primer

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Summary

Most traditional approaches in neo-classical economics are not well suited to dealing with variations in output produced by weather dependent generators and interdependencies between technologies. So far, the issues that arise have been managed on a trial-and-error basis. It is likely, however, that without a better understanding of the logic of the system policies towards zero emissions in electricity generation are building in the wrong models.

In this paper Macroeconomics Advisory attempts to provide the foundations for a more systematic approach. It shows how variations in output affect the growth rates of different technologies along the low emissions trajectory. In the case of Australia, it shows the opportunity cost of forcing a single trajectory pathway.

1 Introduction

Societies are trying to replace high emissions electricity generation technologies with low emissions alternatives but the decisions this requires are difficult because of gaps in our understanding of the economics of this process. One reason for this is that variations in output from weather dependent technologies create supply side disruptions that affect the return to other technologies and the dynamics of the system. These processes are not fully understood in terms of traditional models of supply and demand. The fact that fluctuations in supply create interdependencies across technologies is well known and there is an extensive literature on their effects. Most of this is based on analysis of market and output data, however, and this is tangential to the theoretical interests of this paper. No attempt will be made to give a survey. See items [1] and [3]through [8] for a small sample. In the real world, the issues created by variable outputs are mostly dealt with in pragmatic terms with a mixture of markets and special purpose interventions without much analytical development. This is often based on analogies with traditional approaches in economics. It may be, however, that this is starting from the wrong model.

The purpose of this paper is to fill some of this gap by modelling the effect of variations in supply on the relative growth rates of different technologies as they displace high emissions alternatives. It concentrates on weather dependent and continuous technologies. It asks two questions: [**a**] how do interdependencies between technologies affect trajectories?; [**b**] how do trajectories change as the amount of weather dependent variation changes?

This paper is a first attempt and makes a number of assumptions that are not necessarily consistent with more familiar macro and micro theories. In the longer term it may be possible to get a more unified approach.

It must be stressed that the paper does not consider optimal trajectories. Some of this is done in [2] and [6] for example. It gives the pathways that would be followed if growth in each technology followed a function of instantaneous returns.

Among the main results are that, if the fraction of weather dependent technology is low, its relative rate of growth will be fast. This will decrease as its fraction increases. If variations in output from weather dependent technology are low an increase will increase its growth rate. If the variation in output is high, an increase will decrease its growth rate. For sufficiently high levels of variation the growth rate of the weather dependent technology can go to zero.

In some cases these results could be guessed. Others such as switching between positive and negative growth rates as variation in output increases are a surprise.

2 The model

2.1. The system

Suppose a government has a programme to replace high emissions technology, like coal, oil or gas, with low emissions alternatives. The growth rate of each low emissions technologies will depend on the returns for each unit of investment. This might be understood as the planner's choice variable or as an approximation for investment decisions. The high emissions technology is labelled ξ. Two new technologies are considered. These are labelled ϖ and α. The ϖ technology is solar and wind. This technology is weather dependent. Its capacity is taken as its average output. It fluctuates above and below this to produce a surplus or deficit. The α technology gives continuous output. It includes nuclear and fossil fuels with carbon capture and storage. Total demand for consumption and storage is E. This is constant. The sum of replacement capacity grows until all demand is met. At this point everything stops.

The problem is to find the trajectories of the new technologies and their final shares when all high emissions technologies have been replaced. If there were no interaction between the α and $varpi$ technologies and costs were the same any trajectory from the starting point to the surface E would be satisfactory. See fig. 1 for examples.

The dynamics of the system can be represented by a set of differential equations if we are prepared to accept that interpolating a lumpy system with continuous and almost everywhere differentiable functions is a reasonable approximation. Let the capacity of the ϖ and α technologies at time t be $w'(t)$ and $a'(t)$. This gives

$$\dot{w}' := F(a', w')$$

and

$$\dot{a}' := G(a', w')$$

where $\dot{w}'$ and $\dot{a}'$ are the rate of change in output capacity.

These equations must satisfy the following conditions:

[**a**]. $F = 0$ and $G = 0$ when $E - w' - a' = 0$.

[**b**]. $F + G \geq 0$.

Condition $[a]$ is obvious and $[b]$ says that the old high emissions technology is not allowed to increase.

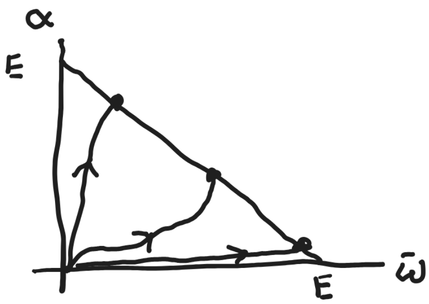

Figure 1. Example of possible trajectories without interdependencies

Perhaps the simplest way to construct these equations is to assume that F can be decomposed into a product of two expressions. The first is common to both technologies and is written $g(E - a' - w')$ with the property that $g(0) = 0$. The second depends on revenue. It is specific to each technology and determines its growth rate. For the ϖ and α technologies respectively these expressions are written $\sigma(a', w')$ and $\psi(a', w')$.

It is now possible to write

$$\dot{w}' = g\sigma \tag{1}$$

$$\dot{a}' = g\psi \tag{2}$$

for all t where it is defined. To save repetition the condition on the region of definition will be assumed.

In order to concentrate on the essential aspects of the problem it is assumed that prices are constant and that the rate of growth of each technology is positively correlated with revenue per unit. This is a crude approach but it has the advantage of mimicking investment trends over long periods of time and gives some idea of how technologies interact in the absence of other variables. It also allows us to avoid dealing with the fact that information is coarse and the long run dynamics are more responsive to average prices and costs than shifts in short term marginal returns.

It is assumed that σ and ψ can be taken as continuous increasing functions of revenue. Brcause price is fixed the primary influence on revenue is demand for output from fixed capacity.

Since the high emissions and the α technology produce a constant output it will sometimes be convenient to consider them jointly. When this is done they are called thermal and written as γ where $\gamma = \xi + \alpha$. Capacity for γ is written x'.

Marginal costs differ for different technologies. For the γ technologies marginal costs are assumed to be greater than zero. For the ϖ technology marginal cost is zero.

Variations in output from the ϖ technology produce a surplus s^+ or a deficit s^- where $s^+ = \int_T s(t)dt$ for a period T with a similar expression for s^-. This is a messy calculation. Fortunately it gives much more precision than is required.

In order to simplify it is assumed that variation can be averaged and treated as a fraction of output. This is written k. It is also assumed that time periods can be standardized and that output is surplus in the period

$(0, \frac{1}{2})$ and negative in the period $(\frac{1}{2}, 1)$. See fig 2.

It makes sense to think of k as an indicator of the way in which storage capacity introduces smoothing into the system. If k is large smoothing capacity is small since fluctuations are not dampened. If k is small fluctuations are dampened. For some sufficiently large K we have $k \to 0$ as *storage capacity* $\to K$.

It is possible for the α and ξ technologies to respond to a deficit by increasing their output by some fraction of total capacity. Let this be $q < 1$. It seems reasonable to treat this as fixed by the technology.

It is assumed that fluctuations in demand for the continuous technology are shared equally across all units. If, for example, the surplus requires a reduction in output of Q, the share for each unit of γ is $\frac{Q}{x'}$.

2.2. Variation in output and revenue functions

The gains or losses from output variations are dealt with by starting with returns for full output and adjusting up or down. Let R be total revenue. Set price as $p = 1$ and adjust for marginal revenues where required. For the ϖ technology

$$R(\varpi) = w' + \frac{1}{2}\upsilon_1(s^+) + \frac{1}{2}\upsilon_2(s^-)$$

where υ_i is a continuous function on its interval of definition. Consider, for example, the case where $\upsilon_1 = \upsilon_2$ and all surplus can be sold and all deficit covered by buying energy. Because $s^+ + s^- = 0$

$$R(\varpi) = w'$$

In the same manner

$$R(\gamma) = x' + \frac{1}{2}(\mu_1(s^+) + \frac{1}{2}\mu_2(s^-)$$

where μ_i is a continuous function on its interval of definition. If variations were symmetrical, or did not affect the γ technology

$$R(\gamma) = x'$$

From this we get the following cases.

[**a**]. The s^+ equations.

The surplus will displace output from the γ technology when it can be sold at any price less than short term marginal cost, written m. This means

$$\upsilon_1(s^+) = mkw' \quad \text{for} \quad kw' \leq x' \quad \text{and} \quad mx' \quad \text{for} \quad kw' \geq x'$$

$$\mu_1(s^+) = -(1-m)kw' \quad \text{if} \quad kw' \leq x' \quad \text{and} \quad -(1-m)x' \quad \text{for} \quad kw' \geq x'$$

[**b**]. The s^- equations.

The deficit after contribution from γ is a cost to ϖ of $(1+\bar{m})$. It is assumed that γ can sell at cost of production $(1+\bar{m})$ but this leaves returns constant since price and costs cancel. Without loss of generality we can set $\bar{m} = 0$. This means that

$$\upsilon_2(s^-) = 0 \quad \text{if} \quad kw' < qx' \quad \text{and} \quad qx' - kw' \quad \text{for} \quad kw' \geq qx'$$

$$\mu_2(s^-) = kw' \quad \text{if} \quad kw' < qx' \quad \text{and} \quad qx' \quad \text{for} \quad kw' \geq qx'$$

It is now possible to rewrite the revenue functions for intervals I_1, I_2 and I_3 given by $kw' < qx'$, $qx' \leq kw' \leq x'$ and $kw' > x'$ as

$$R(\varpi) = w' + \frac{1}{2}mkw';\ w' + \frac{1}{2}(-(1-m)kw' + qx') \quad \text{and} \quad w' + \frac{1}{2}((m+q)x' - kw')$$

$$R(\gamma) = x' + \frac{1}{2}(-(1-m)kw' + kw');\ x' + \frac{1}{2}(-(1-m_1)kw' + qx') \quad \text{and} \quad x' + \frac{1}{2}(-(1-m)x' + qx')$$

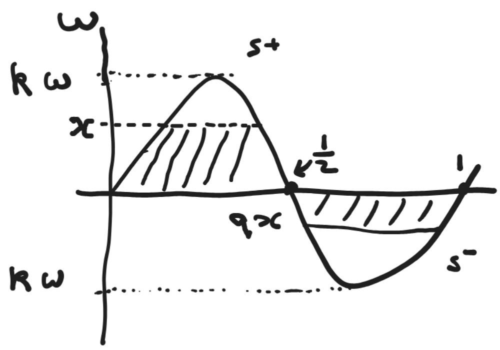

Figure 2. Surplus and deficit

It is unit cost that matter in the dynamics and we also need to get a measure for E since it occurs implicitly in $x' = E - w'$.

It is possible to sort out both issues by defining a new unit of output as $wE := w'$ where w can be thought of as the fraction of total energy produced by ϖ with a similar procedure for x'. Let $r(\varpi) = \frac{R(\varpi)}{Ew}$ and $r(\gamma) = \frac{R(\gamma)}{Ex}$.

On the manifold with $E = 1$ in the (a, w) space where $1 - w - a = 0$ all the old technology is replaced.

To get from revenue equations to the dynamics of the system it is necessary to consider that investment may cease at different lower bounds to revenue. If it is highly sensitive the lower bound might be high. If insensitive it would be low. This is given by c.

From equations (1) and (2) we get $\dot{w}' = E\dot{w}$ and $\dot{a}' = E\dot{a}$. It is possible to write σ and ψ in terms of $\frac{\dot{w}'}{E}$ and $\frac{\dot{a}'}{E}$. Using the fact that $x' = E - w'$ gives

$$\psi = c + \frac{1}{2}(mk(\frac{w}{1-w}));\ c + \frac{1}{2}(-(1-m_1)k(\frac{w}{1-w})) + q) \quad \text{and} \quad c + \frac{1}{2}(-1 + m + q) \tag{3}$$

$$\sigma = c + \frac{1}{2}mk;\ 1 + \frac{1}{2}(-(1-m)k + q\frac{(1-w)}{w}) \quad \text{and} \quad c + \frac{1}{2}((m+q)\frac{(1-w)}{w} - k) \tag{4}$$

across the intervals I_1, I_2 and I_3.

A consequence of this is that the dynamics of the system depend only on w. This is unexpected but easy to understand because of the common characteristics of the γ technology.

In what follows trajectories will be treated as a function of w and the parameter value k. This shows how the behaviour of the system changes for different levels of variability in weather dependent output.

It is assumed that q is fixed by the tchnology and is constant. It is obvious that $q < 1$ and it is also assumed that it is small. This is necessary to preserve the structure of the problem. For q of magnitude close to 1 thermal generation would be back up technology and not of interest

A technical consideration is that the trajectory is not defined in (a, w) space for $k = 0$ since the technologies would be independent. It will be noted, however, that equations (4) and (3) are well behaved and continuous as $k \to 0$ from the right and left.

For simplicity and without loss of generality set $c = 1$.

It is assumed $m < 1$ is small.

3 Trajectories

3.1. Summary of trajectories

The main characteristic of trajectories are, roughly, that the ϖ technology grows rapidly when its share of total generation is low and its rate of growth decreases as its share increases. This is consistent with most observation. If k is small the share of in final output increases as k increases up to some point. After this there is a switch and its share decreases as k increases. This is more of a surprise. It is also possible that at some level of k its growth rate goes to zero. In this case the growth rate in the α technology goes to infinity.

These results are set out more precisely in what follows and fig. 3 gives examples.

3.2. Formal analyis

The relative rates of growth are analyzed in (a, w) phase space. Let $\varphi = \frac{da}{dw}$. Then

$$\varphi = \frac{\psi}{\sigma}$$

and this gives the integral equation

$$\bar{\varphi}(w) = \int_0^w \varphi \tag{5}$$

with $a = \bar{\varphi}(k, w)$ when k is treated as a variable.

Write the junction between intervals $I_i|I_{i+1}$. The integral is properly defined across these intervals because ψ and σ^{-1} are continuous in w and k except at $\sigma = 0$. Continuity at junctions is guaranteed by $q < 1$. Derivatives exist everywhere except across junction points.

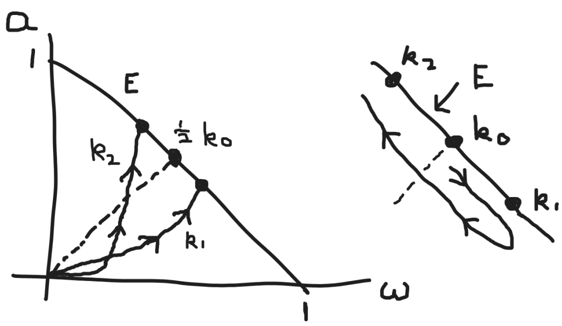

Figure 3. Examples of trajectories. Paths are referred to by the values of k in $\bar{\varphi}$ with $k_2 > 1 > k_1 > k_0 = 0$

Proposition 1 [**a**] The proportions in the energy mix for all k are $w > a$ when w is small and the growth rate for the α technology increases at an increasing rate;

[**b**] w increases in the stationary state given by the surface $1 - a - w = 0$ as k increases for some $k < u$ and decreases for some $k > u$;

[**c**] there is a $k > 1 : \varphi \to \infty$

Proof

[**a**]. The proof is immediate from $\varphi < 1$ in I_1 and $\varphi_w > 0$.

[**b**] The proof is constructed by starting with k arbitrarily close to zero and showing that w on the stationary state increases then decreases.

(i) $k < 1$. In this case $w : I_2|I_3$ is given by $kw = 1 - w$ and $w > \frac{1}{2}$. This means that for $w \leq \frac{1}{2}$ it is only necessary to consider $w \in (I_1, I_2)$.

Consider $k \to 0$. In this case $k < q$ for some fixed $q > 0$ and $w = I_1|I_2 \to 1$ and it is only necessary to consider I_1. For $k \to 0$ we have $w \approx \frac{1}{2}$ in the stationary state and $w - a = o$ where $o \to 0$. Although φ is not defined at $k = 0$ it is OK to take the derivative for k increasing. Leibnitz's rule gives

$$\bar{\varphi}_k = \varphi(k, b(k))\frac{db}{dk} - \varphi(k, c(k))\frac{dc}{dk} + \int_{c(k)}^{b(k)} \frac{\partial\varphi}{\partial k} dw$$

where $w = c(k)$ and $w = b(k)$ are initial and terminal conditions. If $c(k) = 0$ and $b(k)$ is fixed at $w = \frac{1}{2}$ the only term to remaining is

$$\int_{c(k)}^{b(k)} \frac{\partial\varphi}{\partial k} dw = \int_{0}^{b(k)} \frac{m(2w-1)}{(1-w)\sigma^2} dw$$

and since $\bar{\varphi}_k < 0$ a is decreasing as k increases along the surface $w = \frac{1}{2}$.

Evaluating this at $k = 0$ gives a shift down the surface E at $w = \frac{1}{2}$ as Δa. Using a linear approximation with $\varphi = 1$ at $w = \frac{1}{2}$ this shift is $h(|\Delta a|) = sin\theta|\Delta a|$ where $\theta = \frac{\pi}{4}$ and so $h = \frac{|\Delta a|}{\sqrt{2}}$.

By induction this continues for all $k : w \in I_1$ at the stationary state. Since $\varphi > 1$ for $w > \frac{1}{2}$ the angle θ decreases as the stationary value of w increases and hence the increment $w(k_1) - w(k_2)$ for $w \in E$ is decreasing.

From the fact that φ_k has the same expression in I_1 and I_2 and φ_k.

As k increases $w = I_1|I_2$ will decrease and $w \in I_2$ at the stationary state. Since φ_k has the same expression in both states the same argument gives w increasing at a decreasing rate.

(ii). Consider k sufficiently large that $w \in I_3$ for $w < \frac{1}{2}$. For $w = \frac{1}{2}$

$$\bar{\varphi}_k = \int_{0}^{b(k)} \frac{\partial\varphi}{\partial k} dw + \int_{b(k)}^{\frac{1}{2}} \frac{\partial\varphi}{\partial k} dw$$

where $b(k) = I_2|I_3$. In the first integral $\varphi_k < 0$ as before. In the second $\varphi_k = \frac{-\sigma_k\psi}{\sigma^2} > 0$ with $\varphi_{kk} > 0$. As k increases $w = I_2|I_3$ decreases. It follows that there must be some k sufficiently large that the absolute value of the first integral is less than the second and $\bar{\varphi}_k > 0$.

It follows from the continuity of φ that w in the stationary state switches direction. It increases as k increases from $k = \epsilon$ and then decreases for some higher value of k

[**c**]. It is immediate from equation (4) that at $I_1|I_2$ the transition is at $\sigma = 1 - \frac{1}{2}(k(1-(m+q))$ and there is a K sufficiently large that $\sigma \to)$ as $k \to K$. Since the term $f = \frac{1-w}{w}$ in (4) gived $f_w < 0$ We have σ decreasing in w. It follows that there are values for $k < K$ where $\sigma > 0$ at $w = y$ but $\sigma \to 0$ as w increases for $w \in S$. For some fixed k let $\sigma \to 0$ for $w \to W$. It is not possible for $w > W$. If so w is decreasing and $\sigma_w > 0$. This means that the surface $w = W$ is repelling.

□

There are two easy corollaries of this propostion.

Corollary: [**d**] For k sufficiently small $w > a$; [**e**] There is some $k = K$ sufficiently large that $a > w$ for all $k > K$.

Proof: [**d**] is immediate and [**e**] follows immediately from part [**c**] and the continuity of φ.

3.3. Loss function

It is also interesting to calculte the amount of production lost. This is difficult along the pathways but relatively easy in the stationary state. In this case the loss is

$$f_1 = \frac{1}{2}((1-m)kw + (1-q(1-w))kw)$$

when the stationary pont is in I_2 or I_3 with $kw > qa$ and

$$f_2 = \frac{1}{2}(1-m)kw$$

if the stationary point is in I_1.

In the case of f_2 the results are obvious. Differentiating f_1 gives

$$f_{1w} > 0 \quad \text{and} \quad f_{1ww} > 0$$

and the production lost accelerates as w increases.

In both cases the loss of production increases at a linear rate as k increases, as expected.

4 A note weather dependent technologies: Australian application

The model needs to be interpreted for Australia in terms of the opportunity cost of employing the weather dependent technology. It gives a qualitative anlysis of the difference between the amount of weather dependent technology that would be installed to reach a specified share of total energy and the amount that would be installed if the programme followed a trajectory that were determined by rates of change in relative costs. For example, assume that the intention is to reach some fraction $b < 1$ of total energy. For the single technology the path is along the horizontal w until the point b. For some given value of k the instantaneous revenue growth path intercepts the surface b at $w = d$. The difference is $b - d$. See fig. 4.

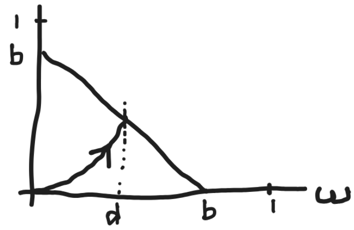

Figure 4. Example of weather dependent technology only

Because the model is dimensionless $b - d$ has no meaning as a number. It is possible, however to assign meaning to relative distance. From the model this will depend on k and the value of b. For example, if only a small b is required a renewables only policy may not have high opportunity costs. These might escalate quickly for high values of b and k.

If k is interpreted as an indication of battery capacity with a small k indicating a large battery capacity, it will be a significant element in the weather dependent technology model, although it will have the same role in the formal analysis. It now has a role in offsetting opportunity cost losses.

5 Remarks

The paper provides a more complete picture of supply side variations and their effect on the dynamics of energy systems than stanadard neo-classical theory. More specifically, it tries to answer questions [**a**] and [**b**] at the beginning by exploring how variations in weather dependent output determine the trajectories in a development programme based on relative returns. Some of the results were, more or less, in keeping with intuitions such as fast initial growth of the weather dependent technology. It was probably more surprising to see that an increase in weather dependent variation initiall accelerated the relative growth of the ϖ technology and later switched to impede it or reduce it to zero. It was also observed that weather dependent technology impose a cost on the system as a whole.

The model is not intended to be realistic, but has a number of implications. Among these are:
(a) A fast initial relative rate of growth in weather dependent technology should not be taken as the basis for predicting continued or accelerated growth;
(b) Responses to changes in technologies or storage capacity may be non-linear;
(c) Attempts to alter the trajectory of part of the system may have unintended consequences for the overall dynamics.
(d) Interactions across the system need to be analyzed in a unified manner. It is a mistake to think of the economics of energy replacement as an aggregate of the economics of single units.

Regarding (a) it was noted that as k increases the relative rate of growth of the weather dependent technology increases in the initial period. It was also noted that a very fast initial rate of growth may indicate a more rapid slow down in later periods.

I am hesitant to fine tune the model by including batteries and other technologies as independent variables and have left them subsumed into the parameter k. This is partly because information on technologies and costs is uncertain. It is also because these additions may come at the expense of insight.

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